

Studies in Nonlinear Dynamics & Econometrics

Volume 13, Issue 2

2009

Article 4

Testing for Conditional Heteroscedasticity in the Components of Inflation

Carmen Broto*

Esther Ruiz[†]

*Banco de España, carmen.broto@bde.es

[†]Universidad Carlos III de Madrid, ortega@est-econ.uc3m.es

Testing for Conditional Heteroscedasticity in the Components of Inflation*

Carmen Broto and Esther Ruiz

Abstract

In this paper we propose a model for monthly inflation with stochastic trend, seasonal and transitory components with QGARCH disturbances. This model distinguishes whether the long-run or short-run components are heteroscedastic. Furthermore, the uncertainty associated with these components may increase with the level of inflation as postulated by Friedman. We propose to use the differences between the autocorrelations of squares and the squared autocorrelations of the auxiliary residuals to identify heteroscedastic components. We show that conditional heteroscedasticity truly present in the data can be rejected when looking at the correlations of standardized residuals while the autocorrelations of auxiliary residuals have more power to detect conditional heteroscedasticity. Furthermore, the proposed statistics can help to decide which component is heteroscedastic. Their finite sample performance is compared with that of a Lagrange Multiplier test by means of Monte Carlo experiments. Finally, we use auxiliary residuals to detect conditional heteroscedasticity in ten monthly inflation series.

*Corresponding author: Carmen Broto. Postal address: Dirección General Adjunta de Asuntos Internacionales, Banco de España, C/Alcalá 48, 28014 Madrid (Spain); Tel: 34 91 3388776. Fax: 34 91 3386212. e-mail: carmen.broto@bde.es. Financial support from project SEJ2006-03919/ECON from the Spanish Government is gratefully acknowledged by the second author. The opinions expressed in this document are sole responsibility of the authors and do not represent the views of the Banco de España or the Eurosystem. The usual disclaimers apply.

1 Introduction

Having accurate measures of inflation uncertainty has become crucial for macroeconomic analysts. Nowadays, it is well accepted that this uncertainty evolves over time. Friedman (1977) suggests that higher inflation levels lead to greater uncertainty about future inflation¹; see Ball (1992) for an economic theory explaining this causality relationship. The empirical evidence on the Friedman hypothesis, also named “*leverage effect*” in the Financial Econometrics literature, is diverse. The first problem faced by the empirical researcher is that the uncertainty of inflation is unobservable and, consequently, there is a question about how to measure it. Early papers used the inflation variability or the forecasts dispersion as proxies for uncertainty; see, for example, Okun (1971), Foster (1978) or Cukierman and Watchel (1979). Later, after the introduction of the ARCH model by Engle (1982), many authors measured the uncertainty of inflation by the conditional variance; see, for example, Engle (1983), Bollerslev (1986) and Cosimano and Jansen (1988). These authors did not find empirical support for the Friedman hypothesis. However, it has been supported by Joyce (1995), Baillie et al. (1996), Grier and Perry (1998), Kim and Nelson (1999), Kontonikas (2004), Conrad and Karanasos (2005) and Daal et al. (2005) among many others. Finally, there are studies as, for example, Hwang (2001), that find a negative relationship between level of inflation and its future uncertainty.

These contradictory results can be explained by taking into account that the GARCH models considered in these papers may have at least one of the following limitations. First, GARCH models assume that the response of the conditional variance to positive and negative inflation changes is symmetric and this property is intrinsically incompatible with the Friedman hypothesis. In this sense, Brunner and Hess (1993) propose a State-Dependent model that allows for asymmetric responses; see also Caporale and McKiernan (1997) for an empirical implementation of this model. Alternatively, Grier et al. (2004) and Daal et al. (2005) also consider modelling the uncertainty of inflation using GARCH models with leverage effect. The second limitation is that some of the models fitted to inflation did not distinguish between short and long run uncertainty. However, papers that made this distinction find stronger evidence of the Friedman hypothesis in the long run although there is mixed evidence; see, for example, Ball and Cecchetti (1990), Evans (1991), Evans and Watchel (1993), Kim (1993), García and Perron (1996), Grier

¹The opposite type of causation, between inflation uncertainty and the level of inflation, has also been considered between others by Cukierman (1992), Fountas *et al.* (2000), Grier et al. (2004) and Conrad and Karanasos (2005) among many others; see Cukierman and Meltzer (1986) for a theoretical justification. However, this relationship has been found to be empirically weaker and we focus on the Friedman hypothesis.

and Perry (1998) and Kontonikas (2004).

The objective of this paper is twofold. First, we propose to represent the dynamic evolution of inflation by an unobserved component model with QGARCH disturbances in order to overcome the above mentioned limitations. The proposed model is able of distinguishing whether the short or the long run components of inflation are heteroscedastic. At the same time, the heteroscedasticity is modelled in such a way that the volatility may respond asymmetrically to positive and negative movements of inflation. Moreover, previous models for monthly inflation have been fitted to seasonally adjusted observations. In our model, the seasonal component is modelled specifically and, consequently, there is no need for a previous seasonal adjustment. In particular, in order to capture the previously mentioned empirical characteristics, we extend the random walk plus noise model with QGARCH disturbances, denoted by Q-STARARCH and proposed by Broto and Ruiz (2006), by adding a homoscedastic seasonal component.

Second, to identify the presence of heteroscedasticity in the components, we propose to use statistics based on the use of the differences between the autocorrelations of squares and the squared autocorrelations of the auxiliary residuals. We analyze the finite sample behaviour of these differences and show that they can be useful to identify conditional heteroscedasticity even in series where looking at the original data or at the traditional standardized residuals, we may conclude that they are homoscedastic. Furthermore, looking at auxiliary residuals may help to identify which of the components is heteroscedastic. However, although a test based on the differences between the autocorrelations of auxiliary residuals is a useful instrument to identify which component is heteroscedastic, the transmission of heteroscedasticity between auxiliary residuals, could generate some ambiguity depending on the particular model generating the data.

The paper is organized as follows. Section 2 introduces the Q-STARARCH model with seasonality and describes its properties. In Section 3, we analyze the finite sample performance of the differences between the sample autocorrelations of squares and the squared autocorrelations of the stationary transformation of the observations and of the standardized residuals. We also analyze these differences for the autocorrelations of auxiliary residuals as a tool to detect whether a given component of the model is conditionally heteroscedastic. Finally, we carry out Monte Carlo experiments to compare the properties of the proposed tests with those of the Lagrange Multiplier (LM) tests proposed by Harvey et al. (1992). In Section 4, the Q-STARARCH model is fitted to monthly inflation series. Finally, Section 5 concludes the paper.

2 Q-STARCH Model with Seasonal Effects

Consider that the series of interest, y_t , can be decomposed into a long run component, representing an evolving level, μ_t , a stochastic seasonal component, δ_t , and a transitory component, ε_t . If the level follows a random walk, the seasonal component is specified using a dummy variable formulation and the transitory component is a white noise, the resulting model for y_t is given by

$$\begin{aligned} y_t &= \mu_t + \delta_t + \varepsilon_t \\ \mu_t &= \mu_{t-1} + \eta_t \\ \delta_t &= - \sum_{i=1}^{s-1} \delta_{t-i} + \omega_t, \end{aligned} \quad (2.1)$$

where s is the seasonal period; see Harvey (1989). The transitory and long-run disturbances are defined by $\varepsilon_t = \varepsilon_t^\dagger h_t^{1/2}$ and $\eta_t = \eta_t^\dagger q_t^{1/2}$ respectively where ε_t^\dagger and η_t^\dagger are mutually independent Gaussian white noise processes and h_t and q_t are defined as QGARCH processes² given by

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \varepsilon_{t-1} \\ q_t &= \gamma_0 + \gamma_1 \eta_{t-1}^2 + \gamma_2 q_{t-1} + \gamma_3 \eta_{t-1}. \end{aligned} \quad (2.2)$$

The parameters $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \gamma_0, \gamma_1, \gamma_2$ and γ_3 satisfy the usual conditions to guarantee the positivity and stationarity of h_t and q_t ; see Sentana (1995). Finally, the disturbance of the seasonal component is assumed to be a Gaussian white noise with variance σ_ω^2 independent of ε_t and η_t . Model (2.1) is able to distinguish whether the possibly asymmetric ARCH effects appear in the permanent and/or in the transitory component. Furthermore, the conditional variances in (2.2) have different responses to shocks of the same magnitude but different sign.

Although the series y_t is non-stationary, it can be transformed into stationarity by taking seasonal differences. The stationary form of model (2.1) is given by

$$\Delta_s y_t = S(L)\eta_t + \Delta \omega_t + \Delta_s \varepsilon_t \quad (2.3)$$

²Alternatively, the variances of the unobserved components can be specified as Stochastic Volatility (SV) processes, as in Stock and Watson (2007). However, the estimation of unobserved component models with SV disturbances is usually based on Simulated Maximum Likelihood and it is rather difficult to extend the method to allow for different components having different evolutions of the volatility; see, for example, Brandt and Kang (2004), Koopman and Bos (2004) and Bos and Shephard (2006). Another proposal of unobserved component models with heteroscedastic errors can be found in Ord *et al.* (1997), where instead of considering different disturbance processes for each unobserved component, the source of randomness is unique.

where Δ_s and Δ are the seasonal and regular difference operators given by $\Delta_s = 1 - L^s$ and $\Delta = 1 - L$ respectively, and $S(L) = 1 + L + \dots + L^{s-1}$. The dynamic properties of $\Delta_s y_t$ can be analyzed by deriving its autocorrelation function (acf) that is given by

$$\rho(h) = \begin{cases} \frac{(s-1)\sigma_\eta^2 - \sigma_\omega^2}{s\sigma_\eta^2 + 2\sigma_\omega^2 + 2\sigma_\varepsilon^2}, & h = 1 \\ \frac{(s-h)\sigma_\eta^2}{s\sigma_\eta^2 + 2\sigma_\omega^2 + 2\sigma_\varepsilon^2}, & h = 2, \dots, s-1 \\ \frac{-\sigma_\varepsilon^2}{s\sigma_\eta^2 + 2\sigma_\omega^2 + 2\sigma_\varepsilon^2}, & h = s \\ 0, & h > s \end{cases} \quad (2.4)$$

where $\sigma_\varepsilon^2 = \alpha_0/(1 - \alpha_1 - \alpha_2)$ and $\sigma_\eta^2 = \gamma_0/(1 - \gamma_1 - \gamma_2)$. The first row of Figure 1 plots the acf in (2.4) for the following four Q-STARCH models with $s = 4$,

	α_0	α_1	α_2	α_3	γ_0	γ_1	γ_2	γ_3	σ_ω^2
M0	1	0	0	0	0.25	0	0	0	0.01
M1	0.05	0.15	0.8	0.17	0.25	0	0	0	0.01
M2	4	0	0	0	0.05	0.15	0.8	0.17	0.01
M3	0.2	0.15	0.8	0.17	0.05	0.15	0.8	0.17	0.01

The values of the parameters have been chosen to resemble the values typically estimated when analyzing real time series of monthly inflation. In particular, the signal to noise ratio of the long run component, $q_\eta = \sigma_\eta^2/\sigma_\varepsilon^2 = 0.25$, is smaller than one, because usually the variance of the long run component of inflation is smaller than the variance of the transitory component. The variance of the seasonal component is also rather small, $\sigma_\omega^2 = 0.01$. With respect to the presence of conditional heteroscedasticity, model *M0* has all its components homoscedastic. However, the short run disturbance of model *M1* is heteroscedastic, while in model *M2* the long run component is heteroscedastic. Finally, both disturbances are heteroscedastic in model *M3*. Note that the acf of $\Delta_4 y_t$ is the same regardless of whether the disturbances are heteroscedastic or homocedastic because the parameters of the QGARCH models have been chosen in such a way that the marginal variance of ε_t and η_t are the same in the four models considered.

Figure 1 also plots the sample means through 1000 replicates of the sample autocorrelations of $\Delta_4 y_t$, $r(h)$, of series of size $T = 200$ generated by the models described before. We can observe that, for the models and sample size considered

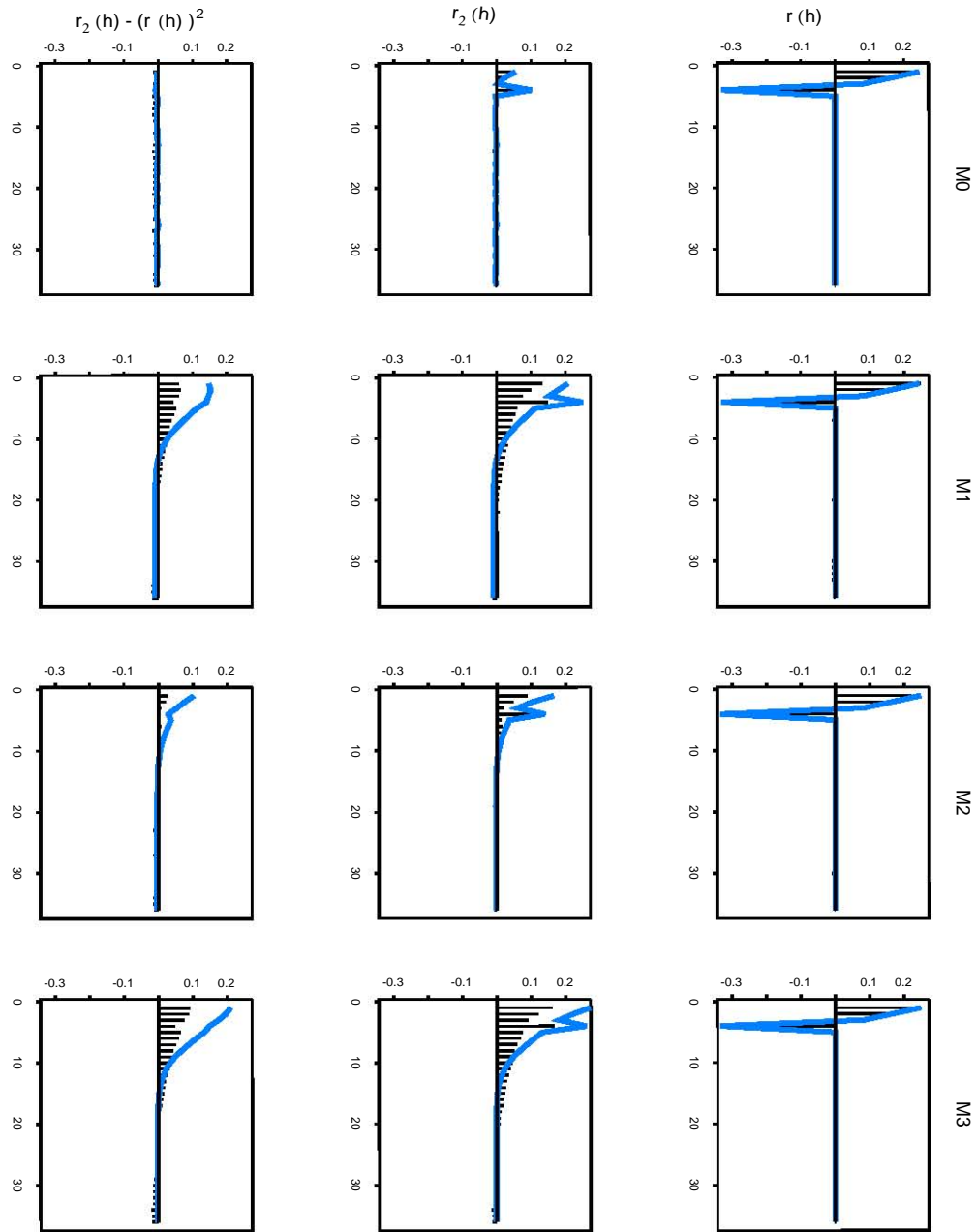


Figure 1: Mean autocorrelation function of (by rows) $\Delta_4 y_t$, $(\Delta_4 y_t)^2$, and differences between the mean autocorrelation function of $(\Delta_4 y_t)^2$ and mean autocorrelation function of $\Delta_4 y_t$ squared for four Q-STARCH models (by columns). Results based on 1,000 replications of series with sample size $T = 200$. Solid lines represent theoretical values and high density lines their sample estimates.

in this illustration, the sample autocorrelations of $\Delta_4 y_t$ are unbiased.

The presence of heteroscedasticity in the model is reflected in the kurtosis of $\Delta_s y_t$ which is given by

$$\begin{aligned} \kappa_y = & (sq_\eta + 2q_\omega + 2)^{-2} \left[q_\eta^2 \left(s\kappa_\eta + 6 \sum_{i=1}^{s-1} (s-i)(1 + (\kappa_\eta - 1)\rho_2^\eta(i)) \right) + \right. \\ & \left. + 2\kappa_\varepsilon + 6(1 + (\kappa_\varepsilon - 1)\rho_2^\varepsilon(s)) + 12(sq_\eta q_\omega + sq_\eta + 2q_\omega + q_\omega^2) \right], \quad (2.5) \end{aligned}$$

where q_ω is the signal to noise ratio of the seasonal component, given by $q_\omega = \sigma_\omega^2 / \sigma_\varepsilon^2$. κ_ε and $\rho_2^\varepsilon(h)$ are the kurtosis and autocorrelations of squares of ε_t , which are given by

$$\kappa_\varepsilon = 3(1 + \alpha_1 + \alpha_2 + (\alpha_3^2 / \alpha_0))(1 - \alpha_1 - \alpha_2) / (1 - 3\alpha_1^2 - \alpha_2^2 - 2\alpha_1\alpha_2) \quad (2.6)$$

and

$$\rho_2^\varepsilon(h) = \begin{cases} \frac{2\alpha_1(1 - \alpha_1\alpha_2 - \alpha_2^2) + (\alpha_3 / \sigma_\varepsilon^2)(3\alpha_1 + \alpha_2)}{2(1 - 2\alpha_1\alpha_2 - \alpha_2^2) + (3\alpha_3 / \sigma_\varepsilon^2)}, & h = 1 \\ (\alpha_1 + \alpha_2)^{h-1} \rho_2^\varepsilon(h-1), & h > 1 \end{cases} \quad (2.7)$$

respectively; see Sentana (1995). The expression of the kurtosis and acf of squares of η_t , κ_η and $\rho_2^\eta(h)$, respectively, are analogous to those of ε_t . As expected, the kurtosis in (2.5) is 3 when all the noises are homoscedastic.

It is well known that when a series is homoscedastic and Gaussian, the autocorrelations of squared observations are equal to the squared autocorrelations of the original observations; see Maravall (1987) and Palma and Zevallos (2004). The presence of conditional heteroscedasticity generates autocorrelations of squares larger than the squared autocorrelations. Consequently, we derive the acf of the squares of the stationary transformation of y_t denoted by $\rho_2(h)$. This acf has been obtained by Broto and Ruiz (2006) for the particular case of the local level model, i.e. model (2.1) without seasonal component. They show that the effect of the presence of asymmetries in the volatilities of the components on the autocorrelations of squares is negligible. Therefore, for simplicity, the asymmetric parameters in equations (2.2), α_3 and γ_3 , are fixed to zero. In this case, after some very tedious although straightforward algebra, we derive the following expression of the

autocovariance function of $(\Delta_s y_t)^2$ in the seasonal Q-STARCh model,

$$\gamma_2(h) = \begin{cases} \sigma_\varepsilon^4 [q_\eta^2 \{ (s-1)(\kappa_\eta - 1) + 2(\kappa_\eta - 1) \sum_{i=1}^{s-1} (s-i) \rho_2^\eta(i) + \rho_2^\eta(s) \\ + 4 \sum_{i=1}^{s-2} (s-i-1)(1 + (\kappa_\eta - 1) \rho_2^\eta(i)) \} + 2q_\omega^2 \\ + (\kappa_\varepsilon - 1) \{ 2\rho_2^\varepsilon(1) + \rho_2^\varepsilon(s-1) + \rho_2^\varepsilon(s+1) \} - 4(s-1)q_\eta q_\omega], & h = 1 \\ \\ \sigma_\varepsilon^4 [q_\eta^2 ((\kappa_\eta - 1) \{ (s-h) + 2(s-h) \sum_{i=1}^{h-1} \rho_2^\eta(i) + 2 \sum_{i=h}^{s-h} (s-i) \rho_2^\eta(i) \\ + \sum_{i=s-h+1}^{s+h-1} (s+h-i) \rho_2^\eta(i) \} + 4 \sum_{i=1}^{s-h-1} (s-i-h)(1 + (\kappa_\eta - 1) \rho_2^\eta(i)) \\ + (\kappa_\varepsilon - 1) \{ 2\rho_2^\varepsilon(h) + \rho_2^\varepsilon(s-h) + \rho_2^\varepsilon(s+h) \}]], & h = 2, \dots, s-1 \\ \\ \sigma_\varepsilon^4 [q_\eta^2 (\kappa_\eta - 1) (\sum_{i=1}^s i \rho_2^\eta(i) + \sum_{i=1}^{s-1} (s+i) \rho_2^\eta(i)) \\ + (\kappa_\varepsilon - 1) \{ 1 + \rho_2^\varepsilon(s) + \rho_2^\varepsilon(2s) \}], & h = s \\ \\ \sigma_\varepsilon^4 [q_\eta^2 (\kappa_\eta - 1) (\sum_{i=h+1-s}^h (i-h+s) \rho_2^\eta(i) + \sum_{i=h+1}^{h+s-1} (h+s-i) \rho_2^\eta(i)) \\ + (\kappa_\varepsilon - 1) \{ 2\rho_2^\varepsilon(h) + \rho_2^\varepsilon(h-s) + \rho_2^\varepsilon(s+h) \}], & h > s. \end{cases} \quad (2.8)$$

The variance of $(\Delta_s y_t)^2$ is given by

$$\begin{aligned} Var [(\Delta_s y_t)^2] &= \sigma_\varepsilon^4 \left[q_\eta^2 \left(2s^2 + s(\kappa_\eta - 3) + 6(\kappa_\eta - 1) \sum_{i=1}^{s-1} (s-i) \rho_2^\eta(i) \right) + \right. \\ &\quad \left. 8q_\omega^2 + 2(\kappa_\varepsilon + 1) + (4(\kappa_\varepsilon - 1) \rho_2^\varepsilon(s)) + 8sq_\eta q_\omega + 8sq_\eta + 12q_\omega \right]. \end{aligned} \quad (2.9)$$

From expression (2.8) and (2.9), it is possible to obtain the expression of the acf of $(\Delta_s y_t)^2$. Note that when the signal to noise ratio of the long-run component is small, as in the case of inflation, the heteroscedasticity of this component does not affect the autocorrelations of squares. However, when this ratio is large, the effect of a heteroscedastic long-run component is larger than the effect of the transitory component. This result is illustrated in the second row of Figure 1 that plots the acf of $(\Delta_s y_t)^2$ for the same four models considered above.

The third row of Figure 1 plots the population differences $\rho_2(h) - (\rho(h))^2$. Note that in the homoscedastic model $M0$, the autocorrelations of squares are similar to the squared autocorrelations of $\Delta_s y_t$. However, in the $M1$ and $M3$ models in which the transitory component is heteroscedastic, the autocorrelations of $(\Delta_s y_t)^2$ are clearly larger than those of $\Delta_s y_t$. Finally, in model $M2$ the autocorrelations of

squares are only slightly larger than the squared autocorrelations. Note that, because σ_ε^2 is larger than σ_η^2 , the characteristics of the short run component are expected to be more evident in the reduced form than those of the long run component.

To analyze the finite sample properties of the estimates of the autocorrelations of $(\Delta_s y_t)^2$, Figure 1 also plots the sample means through 1000 replicates of the sample autocorrelations of $(\Delta_4 y_t)^2$, $r_2(h)$, of series of size $T = 200$ generated by each of the models. We can observe that the biases of the sample autocorrelations of $(\Delta_4 y_t)^2$ are negative for small lags and slightly positive for large lags. It is important to note that in the case of model $M2$ it could be difficult to detect the presence of conditional heteroscedasticity by looking at the differences between the autocorrelations of $(\Delta_4 y_t)^2$ and the squared autocorrelations of $\Delta_4 y_t$.

3 Testing for Heteroscedasticity

Given that conditional heteroscedasticity generates autocorrelations of squares larger than squared autocorrelations, one can test for it by testing whether the differences between both statistics are significantly larger than zero. As far as we know, the asymptotic properties of these differences are unknown. Therefore, in this section, we analyze by means of Monte Carlo experiments whether they can be approximated by a Normal distribution. We show that looking at the differences between the autocorrelations of $(\Delta_s y_t)^2$ and the squared autocorrelations of $\Delta_s y_t$, the heteroscedasticity can be rejected when it is truly present in the data. We also analyze the finite sample properties of the differences between the autocorrelations of the innovations. In this case, the asymptotic distribution is known as their autocorrelations are zero. Finally, we look at the autocorrelations of auxiliary residuals. The properties of the proposed tests are compared with those of the LM tests.

3.1 Tests based on the stationary transformation

As we mentioned above, when the noises of model (2.1) are homoscedastic and Gaussian, the autocorrelations of $(\Delta_s y_t)^2$ are equal to the squared autocorrelations of $\Delta_s y_t$ in expression (2.4). However, in the presence of heteroscedasticity, the autocorrelations of squares are larger than the squared autocorrelations. Therefore, one can use the differences between both quantities to identify whether a series is heteroscedastic. In this subsection, we analyze the finite sample distribution of $r_2(h) - (r(h))^2$ by Monte Carlo simulations. First row of Figure 2 plots the QQ-plots corresponding to these differences calculated using 1000 replicates simulated by the same models as above when $T = 10000$. This figure shows that when the series is homoscedastic, i.e. both disturbances are homoscedastic, the asymptotic

distribution of $r_2(1) - (r(1))^2$ can be adequately approximated by a $N(0, 1/\sqrt{T})$ distribution for large sample sizes.

Consequently, we propose to test the joint null of $H_0 : \rho_2(h) - (\rho(h))^2 = 0$ for $h = 1, \dots, M$ versus the alternative that at least one of these differences is larger than zero using the following statistic

$$BP_y(M) = T \sum_{h=1}^M (r_2(h) - (r(h))^2)^2. \quad (3.1)$$

Given that under the null hypothesis, $r_2(h) - (r(h))^2$ can be approximated by a $N(0, 1/T)$ distribution in large samples, the distribution of the statistic in (3.1) can be approximated by a χ_M^2 distribution. The finite sample size and size-adjusted power of the test in (3.1) have been analyzed generating 10000 replicates from the same models considered before. The results are represented for a nominal size of 5%. The sizes (results corresponding to model $M0$) and powers (results corresponding to $M1$, $M2$ and $M3$) for $M = 1, 4, 12$ and 24 appear in Table 1, when $T = 100, 200$ and 500 . These results show that for the sample sizes and models considered, $M = 12$ is a good compromise between size and power. Furthermore, Table 1 shows that the power in model $M2$ is very low in concordance with the results illustrated in previous section in Figure 1.

3.2 Tests based on standardized residuals

The test above is based on testing whether the stationary transformation, $\Delta_s y_t$, is homoscedastic. Alternatively, it is possible to test for conditional homoscedasticity by looking at the autocorrelations of squared innovations, $\nu_t^2 = (\Delta_s y_t - E_{t-1}(\Delta_s y_t))^2$. The $t - 1$ in the expectation operator means that the expectation is conditional on the information available at time $t - 1$. The innovations are uncorrelated and consequently, if we want to test for homoscedasticity, we have to look at whether the autocorrelations of ν_t^2 , denoted by $\rho_2^\nu(h)$, are zero or not. Consider now model (2.1) with homoscedastic disturbances. If the parameters were known, the Kalman filter generates Minimum Mean Square Linear (MMSL) one-step ahead prediction errors; see, for example, Harvey (1989). Therefore, running the Kalman filter with estimated parameters³, we can obtain estimates of the innovations, $\hat{\nu}_t$, and compute the autocorrelations of their squares. The first row of Figure 3 plots, for the same models as before, the average differences between these autocorrelations and the corresponding squared autocorrelations obtained assuming that the

³In this paper, we estimate the parameters using the QML estimator proposed by Harvey et al. (1992). Analytical expressions of the Kalman filter for the case of the Q-STARCH model with seasonal effects and the FORTRAN codes employed in the estimation are available upon request.

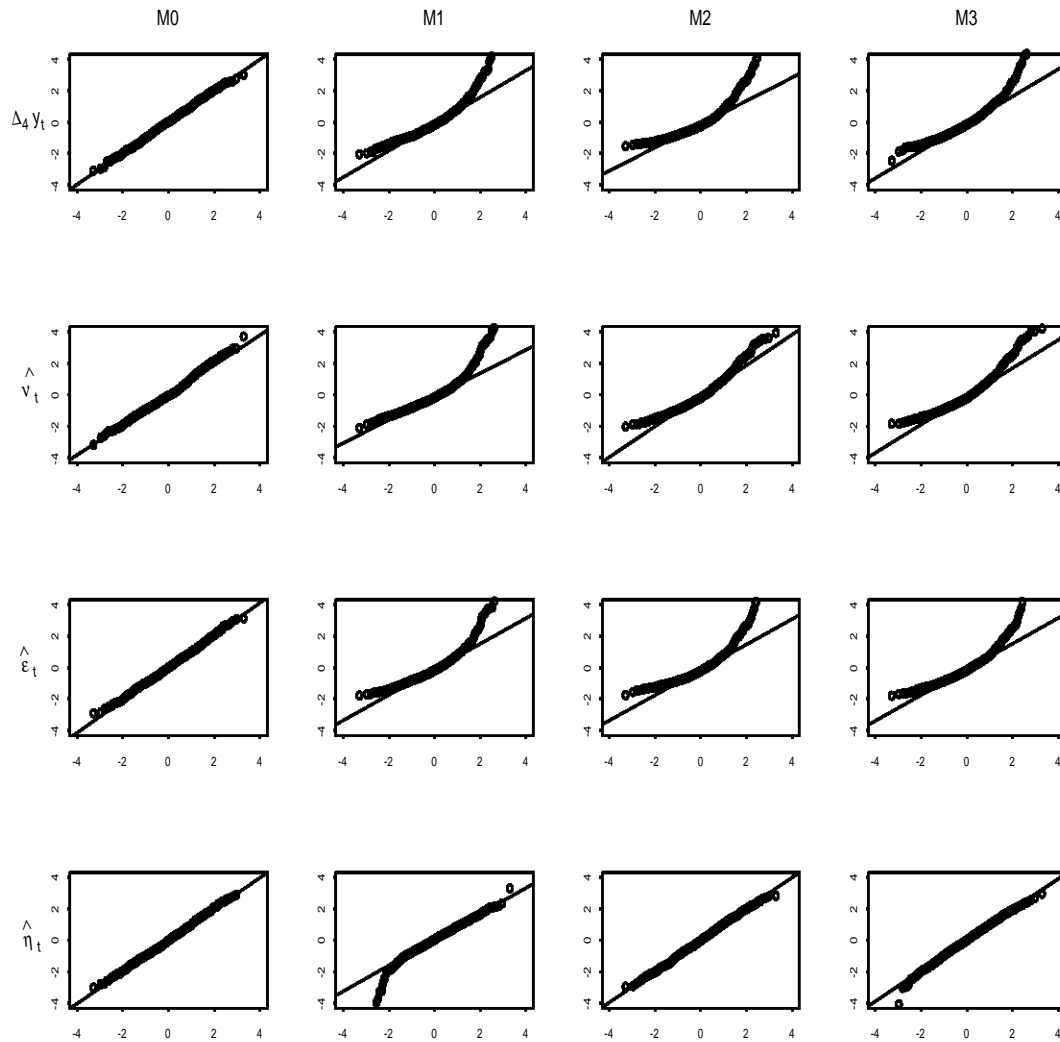


Figure 2: QQ-plots of the differences between the first order autocorrelation of $(\Delta_4 y_t)^2$ and the squared autocorrelation of $\Delta_4 y_t$, and analogue results for \hat{v}_t , $\hat{\varepsilon}_t$ and $\hat{\eta}_t$.

		$T = 100$				$T = 200$				$T = 500$			
		$M = 1$	$M = 4$	$M = 12$	$M = 24$	$M = 1$	$M = 4$	$M = 12$	$M = 24$	$M = 1$	$M = 4$	$M = 12$	$M = 24$
$M0$	$\Delta_4 y_t$	0.0487	0.0475	0.0314	0.0229	0.0607	0.0629	0.0494	0.0402	0.0786	0.0841	0.0666	0.0629
	$\hat{\nu}_t$	0.0399	0.0388	0.0358	0.0229	0.0437	0.0436	0.0414	0.0371	0.0455	0.0465	0.0480	0.0432
	$\hat{\varepsilon}_t$	0.0523	0.0452	0.0381	0.0247	0.0526	0.0521	0.0486	0.0356	0.0603	0.0615	0.0582	0.0472
	$\hat{\eta}_t$	0.0800	0.1080	0.1328	0.1999	0.0931	0.1494	0.1813	0.2002	0.1083	0.1809	0.2300	0.2786
$M1$	$\Delta_4 y_t$	0.0992	0.1398	0.1175	0.0730	0.2101	0.3255	0.3352	0.2861	0.4489	0.6871	0.7177	0.6922
	$\hat{\nu}_t$	0.1128	0.1671	0.1450	0.0828	0.2273	0.3947	0.4140	0.3462	0.5741	0.7803	0.8179	0.7774
	$\hat{\varepsilon}_t$	0.1467	0.2271	0.1976	0.1210	0.3363	0.5327	0.5207	0.4477	0.7508	0.9027	0.9210	0.8887
	$\hat{\eta}_t$	0.0876	0.1482	0.1621	0.1410	0.1271	0.2416	0.3076	0.3189	0.1693	0.4034	0.5172	0.5420
$M2$	$\Delta_4 y_t$	0.0709	0.0771	0.0606	0.0399	0.1423	0.1629	0.1588	0.1321	0.3097	0.3564	0.3610	0.3409
	$\hat{\nu}_t$	0.0568	0.0607	0.0503	0.0309	0.0978	0.1106	0.1123	0.0915	0.1906	0.2225	0.2357	0.2171
	$\hat{\varepsilon}_t$	0.0540	0.0518	0.0474	0.0280	0.0627	0.0726	0.0722	0.0610	0.0908	0.1142	0.1242	0.1145
	$\hat{\eta}_t$	0.0902	0.1249	0.1347	0.1191	0.1884	0.2648	0.3025	0.3080	0.4594	0.5640	0.6125	0.6265
$M3$	$\Delta_4 y_t$	0.1250	0.1773	0.1550	0.0973	0.3018	0.4337	0.4491	0.3844	0.6623	0.8390	0.8708	0.8589
	$\hat{\nu}_t$	0.1444	0.1985	0.1763	0.1083	0.3146	0.4766	0.4943	0.4245	0.6998	0.8793	0.9051	0.8727
	$\hat{\varepsilon}_t$	0.1652	0.2305	0.2088	0.1297	0.3575	0.5228	0.5326	0.4639	0.7448	0.9098	0.9272	0.9022
	$\hat{\eta}_t$	0.0921	0.1588	0.1823	0.1569	0.1978	0.3682	0.4393	0.4287	0.4398	0.7348	0.8144	0.8140

Table 1: Size and power of the test based on a $BP(M)$ statistic build from the difference between the autocorrelation of $(\Delta_4 y_t)^2$ and the squared autocorrelation of $\Delta_4 y_t$, and analogue results for $\hat{\nu}_t$, $\hat{\varepsilon}_t$ and $\hat{\eta}_t$.

parameters are known and $T = 10,000$ together with the average differences obtained with estimated parameters and $T = 200$. First, we can observe important negative biases. Furthermore, comparing the differences of the innovations with the differences corresponding to $\Delta_4 y_t$, we can observe that both plots have similar shapes. The only noticeable difference is that the differences of the innovations are slightly larger. However, the autocorrelations of squared innovations of model $M2$ still do not allow identifying the heteroscedasticity.

The autocorrelations of squared residuals are, under the null, asymptotically distributed with a $N(0, 1/T)$ distribution. Therefore, as before, we can test for conditional homoscedasticity by testing the null hypothesis $H_0 : \rho_2^\nu(1) = \dots = \rho_2^\nu(M) = 0$ versus the alternative that at least one of the autocorrelations is larger than zero by using the following statistic

$$BP_\nu(M) = T \sum_{h=1}^M (r_2^\nu(h))^2. \quad (3.2)$$

Table 1 reports the Monte Carlo sizes and size-adjusted powers of the test statistic (3.2) when the data is generated by models $M0$, $M1$, $M2$ and $M3$, for $M = 1, 4, 12$ and 24 , with $T = 100, 200$ and 500 . Looking at the results reported for model $M0$, we can observe that the size is approximately equal to the nominal for moderate sample sizes and when $M = 12$. On the other hand, in models $M1$ and $M3$, the power increases with respect to testing for heteroscedasticity by looking at the differences between the autocorrelations of $\Delta_s y_t$. However, the power is reduced in model $M2$ in which the long-run component is heteroscedastic.

Summarizing, looking at the differences between the autocorrelations of squares and the squared autocorrelations of $\Delta_s y_t$ and $\hat{\nu}_t$ could be an instrument to detect conditional heteroscedasticity in the disturbances of unobserved component models. However, tests based on these differences may have low power mainly when the heteroscedasticity appears in components with small signal to noise ratio. There are cases, as, for example, model $M2$, in which we can erroneously conclude that the model is homoscedastic. Koopman and Bos (2004), looking at alternative statistics to detect conditional heteroscedasticity in the innovations, also conclude that these statistics have low power. Furthermore, even when these differences are not zero, as in models $M1$ and $M3$, they do not allow us to identify whether the heteroscedasticity affects the long run, the short run or both. Next, we analyze how to use the auxiliary residuals to solve these problems.

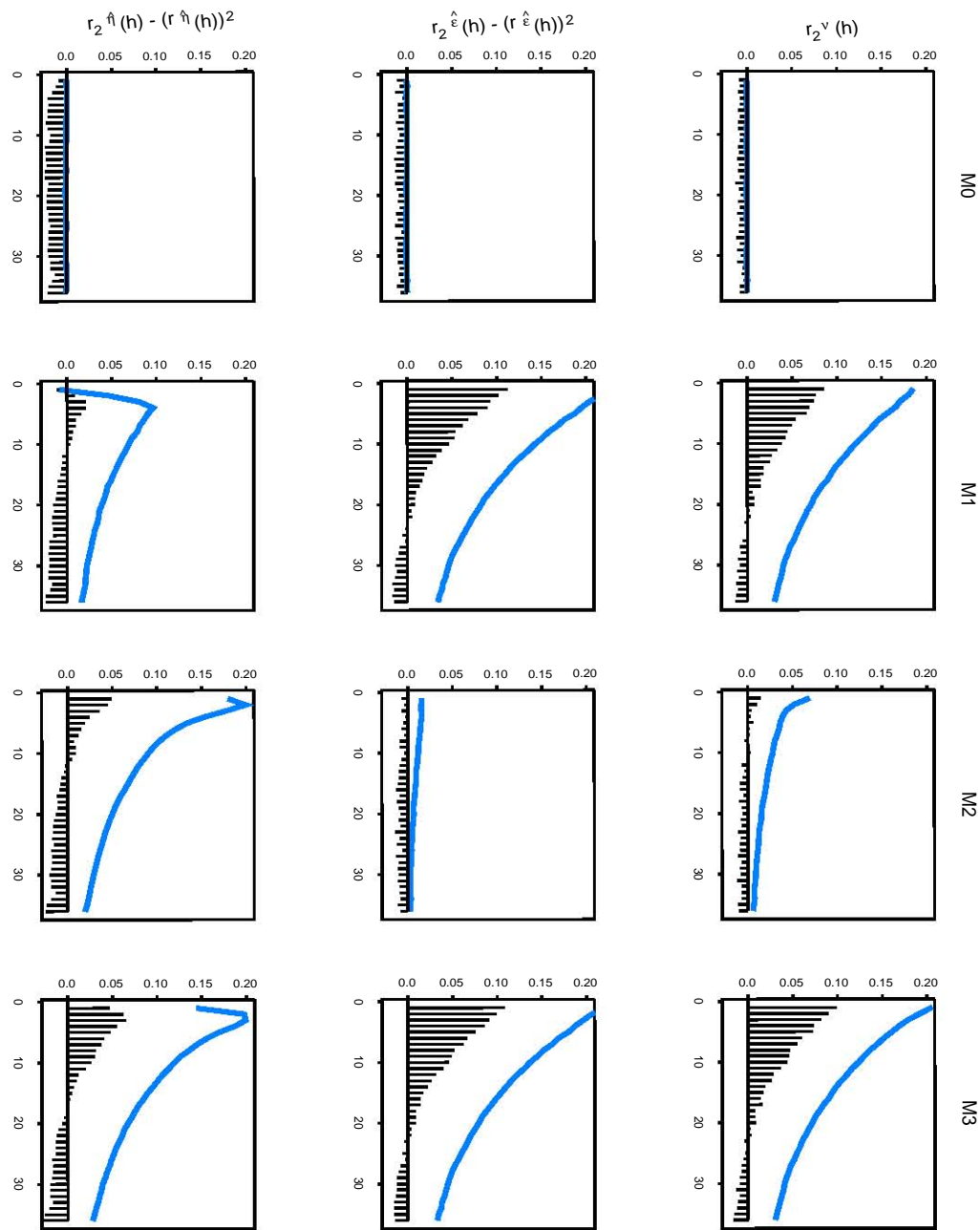


Figure 3: Mean autocorrelation function of $(\hat{\nu}_t)^2$ minus mean squared autocorrelation function of $\hat{\nu}_t$, and analogue results for $\hat{\varepsilon}_t$ and $\hat{\eta}_t$ (by rows) for four Q-STARCH models (by columns). Results are based on 1,000 replications. The continuous lines represent results for sample size $T = 10000$ and known parameters while the high density lines represent results for sample size $T = 200$.

3.3 Tests based on auxiliary residuals

In unobserved component models, it can also be useful to analyze the auxiliary residuals, which are estimates of the disturbances of each component. Harvey and Koopman (1992) derive the expressions of the auxiliary residuals, $\hat{\varepsilon}_t$, $\hat{\eta}_t$ and $\hat{\omega}_t$ which are defined as the MMSL smoothed estimators of ε_t , η_t and ω_t , respectively; see also Durbin and Koopman (2001). In particular, the auxiliary residuals corresponding to model (2.1) are given by

$$\begin{aligned}\hat{\varepsilon}_t &= \frac{(1 - F^s)}{\theta(F)} \frac{\sigma_\varepsilon^2}{\sigma^2} \xi_t \\ \hat{\eta}_t &= \frac{(1 - F^s)}{\theta(F)(1 - F)} \frac{\sigma_\eta^2}{\sigma^2} \xi_t \\ \hat{\omega}_t &= \frac{(1 - F)}{\theta(F)} \frac{\sigma_\omega^2}{\sigma^2} \xi_t,\end{aligned}\tag{3.3}$$

where F is the lead operator such that $Fx_t = x_{t+1}$, $\theta(F)$ is a polynomial of order $s+1$, ξ_t is the reduced form disturbance and σ^2 its corresponding variance. The reduced form disturbance is the unique disturbance of the ARIMA representation of y_t . In particular, the reduced form of model (2.1) is an $ARIMA(0, 0, s) \times (0, 1, 1)_s$ model; see Harvey (1989). Due to the presence of heteroscedasticity in the components, the innovations of the reduced form of $\Delta_s y_t$ are uncorrelated although not independent neither Gaussian; see Breidt and Davis (1992). The non-Gaussianity and the lack of independence may affect the sample properties of some estimators often used in empirical applications.

We propose to use the autocorrelations of auxiliary residuals to identify which disturbances of an unobserved components model are heteroscedastic⁴. Once more, the identification is based on whether the differences between the autocorrelations of squares and the squared autocorrelations of each auxiliary residual are different from zero.

The acf of the auxiliary residuals can be obtained from the expressions in Durbin and Koopman (2001). However, the expressions of the acf of the squared auxiliary residuals are not easy to obtain. Consequently, we use simulated data to analyze the usefulness of the auxiliary residuals to identify heteroscedasticity in the disturbances of seasonal unobserved components models. We have generated 1000 replicates of size $T = 10,000$ by models $M0$, $M1$, $M2$ and $M3$. Figure 3 plots the Monte Carlo means of the differences between the autocorrelations of $\hat{\varepsilon}_t^2$ and $\hat{\eta}_t^2$ and the squared autocorrelations of $\hat{\varepsilon}_t$ and $\hat{\eta}_t$ when the auxiliary residuals have

⁴Wells (1996) proposed to use recursive residuals of the transitory component to test for heteroscedasticity; see Bhar and Hamori (2004).

been obtained assuming that the model parameters are known. This figure shows that in the homoscedastic model, $M0$, none of the auxiliary residuals have autocorrelations of squares larger than the squared autocorrelations. On the other hand, the results for model $M3$ show clearly that the transitory and long-run components are heteroscedastic. The results for model $M2$ also indicate that the long-run component is heteroscedastic while the transitory component is homoscedastic. However, in model $M1$, even though the heteroscedasticity is much more evident in the short-run component than in the long-run component, the differences $r_2^{\hat{\eta}}(h) - (r^{\hat{\eta}}(h))^2$ are different from zero. This could be due to the fact that σ_ε^2 is four times larger than σ_η^2 and, therefore, the heteroscedasticity of ε_t is somehow transmitted to $\hat{\eta}_t$. On the other hand, in this case, when η_t is heteroscedastic, there is not transmission towards $\hat{\varepsilon}_t$ ⁵.

Figure 3 also plots the differences between the squared autocorrelations and the autocorrelations of squares of the auxiliary residuals when they are estimated using the QML estimates of the parameters instead of the true parameters and the sample size is $T = 200$. Although the differences are negatively biased when the estimated parameters are used in the smoothing algorithm, the same patterns can be observed regardless of whether the parameters are known or estimated. Therefore, the differences between autocorrelations of auxiliary residuals seem to help to identify which disturbance is heteroscedastic. Furthermore, the transmission of heteroscedasticity between auxiliary residuals is smaller than when using the true parameters to run the filters.

Given that to the best of our knowledge, the asymptotic distribution of the differences between the autocorrelations of squares and the squared autocorrelations is unknown, we have checked whether it can be approximated by a $N(0, 1/T)$ distribution by means of Monte Carlo experiments. We have simulated 1000 series of size $T = 10,000$. The QQ-plots corresponding to $Corr[\hat{\varepsilon}_t^2, \hat{\varepsilon}_{t-1}^2] - (Corr[\hat{\varepsilon}_t, \hat{\varepsilon}_{t-1}])^2$ and $Corr[\hat{\eta}_t^2, \hat{\eta}_{t-1}^2] - (Corr[\hat{\eta}_t, \hat{\eta}_{t-1}])^2$ appear in Figure 2 for the four models considered above. These plots show that the asymptotic distribution of the differences between autocorrelations of the auxiliary residuals can be approximated by a $N(0, 1/T)$ distribution when the model is homoscedastic. However, when there is heteroscedasticity in at least one of the components, the differences between the autocorrelations corresponding to the transitory disturbance, ε_t , lose the normality especially in the positive tail. However, the distribution of the differences corresponding to the long-run disturbance, η_t , is close to normality in the positive tail although there are deviations in the left tail.

We have also analyzed the finite sample sizes and size-adjusted powers of the

⁵Harvey et al. (1992) also observe some transmission of heteroscedasticity between components when using LM tests to identify which component is heteroscedastic.

$BP(M)$ statistic in (3.1) when implemented to test whether the first M differences between autocorrelations of ε_t and η_t are jointly equal to zero. The results are reported in Table 1. First, observe that the size of the statistic when implemented to test for conditional homoscedasticity in ε_t is adequate in model $M0$ and slightly longer than the nominal in model $M2$ in which the long-run component is heteroscedastic. However, the results for η_t show that the test is always oversized. Note that even in the homoscedastic model, the test BP reject the homoscedasticity of η_t more often than it should do. The oversize is even worse in $M1$ due to the transmission of volatility. When looking at the size-adjusted powers, we can observe that they increase when the test is implemented to test for the homoscedasticity of the auxiliary residuals with respect to testing for the homoscedasticity of $\Delta_s y_t$ or ν_t .

Finally, we have computed the percentage of correct identifications of the model when $T = 500$ and $M = 12$, i.e. of rejecting the null of homoscedasticity when the component is truly heteroscedastic while not rejecting when the component is homoscedastic. This percentage is rather large, around 75% and 73%, in models $M3$ and $M0$ respectively. However, it decreases in models $M2$ and $M1$ when the heteroscedastic components are correctly identified in 53% and 44% of the simulated series.

3.4 Comparison with LM tests

Harvey et al. (1992) propose to test for conditional heteroscedasticity in the components of unobserved component models by using the Lagrange Multiplier (LM) principle. In this subsection, we compare the finite sample size and power of the LM tests with the corresponding tests based on squared autocorrelations described above. The LM test statistic for homoscedasticity is constructed from the uncentered coefficient of determination, R^2 , of a regression of ν_j^* on \mathbf{x}_j , where ν_j^* and the $n \times 1$ vector \mathbf{x}_j , with n equal to the number of parameters, are defined for $j = 1, \dots, 2T$ by the following expressions

$$\begin{aligned}\nu_t^* &= 2^{-1/2} \left[1 - \left(\frac{\nu_t^2}{f_t} \right) \right], \quad t = 1, \dots, T \\ \nu_{T+t}^* &= \nu_t f_t^{-\frac{1}{2}}, \quad t = 1, \dots, T\end{aligned}\tag{3.4}$$

and

$$\begin{aligned}\mathbf{x}_t &= \frac{1}{\sqrt{2}} \frac{1}{f_t} \frac{\partial f_t}{\partial \Psi}, \quad t = 1, \dots, T \\ \mathbf{x}_{T+t} &= \frac{1}{f_t^{1/2}} \frac{\partial \nu_t}{\partial \Psi}, \quad t = 1, \dots, T\end{aligned}\tag{3.5}$$

where Ψ is the parameter vector and f_t the corresponding variance of innovations ν_t , both computed by the Kalman filter. Both ν_t^* and \mathbf{x}_t are evaluated under the null hypothesis; see Harvey (1989, pp. 240-241). Results for the LM test are reported in Table 2. Comparing the size of the LM test with this of the $BP(12)$ test based on the innovations, ν_t , we can observe that the latter is closer to the nominal than the former although both are comparable. Furthermore, the power of the $BP(12)$ test based on ν_t is clearly larger than the power of LM in moderate and large sample sizes.

	LM			$LM(\varepsilon)$			$LM(\eta)$		
	$T = 100$	$T = 200$	$T = 500$	$T = 100$	$T = 200$	$T = 500$	$T = 100$	$T = 200$	$T = 500$
$M0$	0.0337	0.0378	0.0415	0.1336	0.1544	0.1632	0.1244	0.1428	0.1495
$M1$	0.2568	0.2956	0.3245	0.3803	0.4519	0.5770	0.2528	0.3227	0.4897
$M2$	0.0878	0.0983	0.1068	0.1075	0.1305	0.1531	0.8525	0.9697	0.9992
$M3$	0.3073	0.3331	0.3535	0.3333	0.3812	0.4065	0.1940	0.3323	0.4207

Table 2: Size and power of the test based on LM principle.

This test can also be conveniently transformed to test the null hypothesis of homoscedasticity in the transitory component, that is, $H_0 : \alpha_1 = 0$ or in the permanent component, $H_0 : \gamma_1 = 0$. Both tests will be denoted as $LM(\varepsilon)$ and $LM(\eta)$, respectively; see Harvey et al. (1992) for the expressions of these tests. Table 2 also reports the finite sample sizes and size-adjusted powers of both tests when implemented in the same four models considered before. Comparing the sizes of the $LM(\varepsilon)$ test with those of the $BP_\varepsilon(12)$ reported in Table 1, we can observe that the test $LM(\varepsilon)$ is oversized even in model $M0$ and that the distortion of its size is larger in model $M2$ than the one observed for $BP_\varepsilon(12)$. However, when testing for the homoscedasticity of η_t , the sizes of both tests are comparable in model $M1$, while the size of $LM(\eta)$ is slightly close to the nominal in model $M0$. When comparing the power of both tests for testing for conditional homoscedasticity in the transitory component, we observe that in moderate and large samples, the test based on squared autocorrelations has larger power. On the other hand, the power of the $LM(\eta)$ test is larger in the $M2$ model when only the long-run component is heteroscedastic, while it is smaller in the $M3$ model. Overall, it seems that the properties of the tests for the homoscedasticity of the transitory components are better when the $BP_\varepsilon(M)$ test is implemented while depending on the particular model, the $LM(\eta)$ test may be better to test for homoscedasticity in the long-run.

Finally, we analyze the percentage of correct identifications of the model when $T = 500$ and the LM tests are implemented. Although this share is rather large, it only outperforms the results of the proposed BP test for $M2$ model where it

is 78%. For the rest of the models, the percentages of correct identifications are higher for *BP* than for *LM*, as in models *M0*, *M1* and *M3* the later are 71%, 38% and 35%, respectively. That is, *BP* test outperforms *LM* test in most cases, even when a small signal to noise ratio makes it difficult the identification of the heteroscedasticity in the long-run component.

4 Empirical Analysis

In this section, monthly inflation series of eight developed countries (France, Germany, Italy, Japan, the Netherlands, Spain, Sweden and United Kingdom) and two emerging countries (Colombia and Thailand) are analyzed by means of the previously proposed Q-STARCH model. In particular, we have data on inflation measured as first differences of the monthly CPI, i.e., $y_t = 100 * \Delta \log(CPI_t)$. Sample sizes are $T = 513$ for the set of developed countries, from January 1962 until September 2004, whereas for Colombian inflation $T = 556$, running from January 1962 to May 2008, and from January 1970 to May 2008 in the case of Thailand.⁶ Figure 4 plots the ten series of inflation, y_t , together with the differences between the autocorrelations of $(\Delta_{12}y_t)^2$ and the squared autocorrelations of $\Delta_{12}y_t$. Note that the autocorrelations of squares are clearly larger than the squared autocorrelations of the levels for Italy, Japan, the Netherlands, Spain and Thailand, suggesting that these series may be conditionally heteroscedastic. For inflation series of the rest of the countries, this evidence is not so conclusive. Furthermore, all the series have kurtosis coefficients significantly greater than 3 which run from 4.62 for Japan up to 8.16 for France, so they seem to have non-Gaussian distributions.

We start by fitting model (2.1) with homoscedastic disturbances to each of the inflation series. The estimated parameters appear in Table 3. First of all, note that for all inflation series, the estimates of the signal to noise ratios of the long-run component are very small running from 0.007 for Sweden to 0.268 for Colombia. Furthermore, the variances of the seasonal components are also rather small when compared with the variance of the transitory component. Figure 4 plots the estimated long-run components and Figure 5 plots the seasonal components for each of the series of inflation. Note that the seasonal components of France and Italy could be well approximated by assuming that they are deterministic. However, the results for these two countries obtained assuming deterministic seasonality are similar and, therefore, we report the results obtained for stochastic seasonality. Table

⁶Prior to their analysis, the series have been filtered to be rid of outliers. To detect outliers in the different components we have used the detection method of Harvey and Koopman (1992) as implemented in the program STAMP 6.20; see Koopman et al. (2000). The outliers detected affect mainly the transitory component although we found level outliers in Italy and the Netherlands.

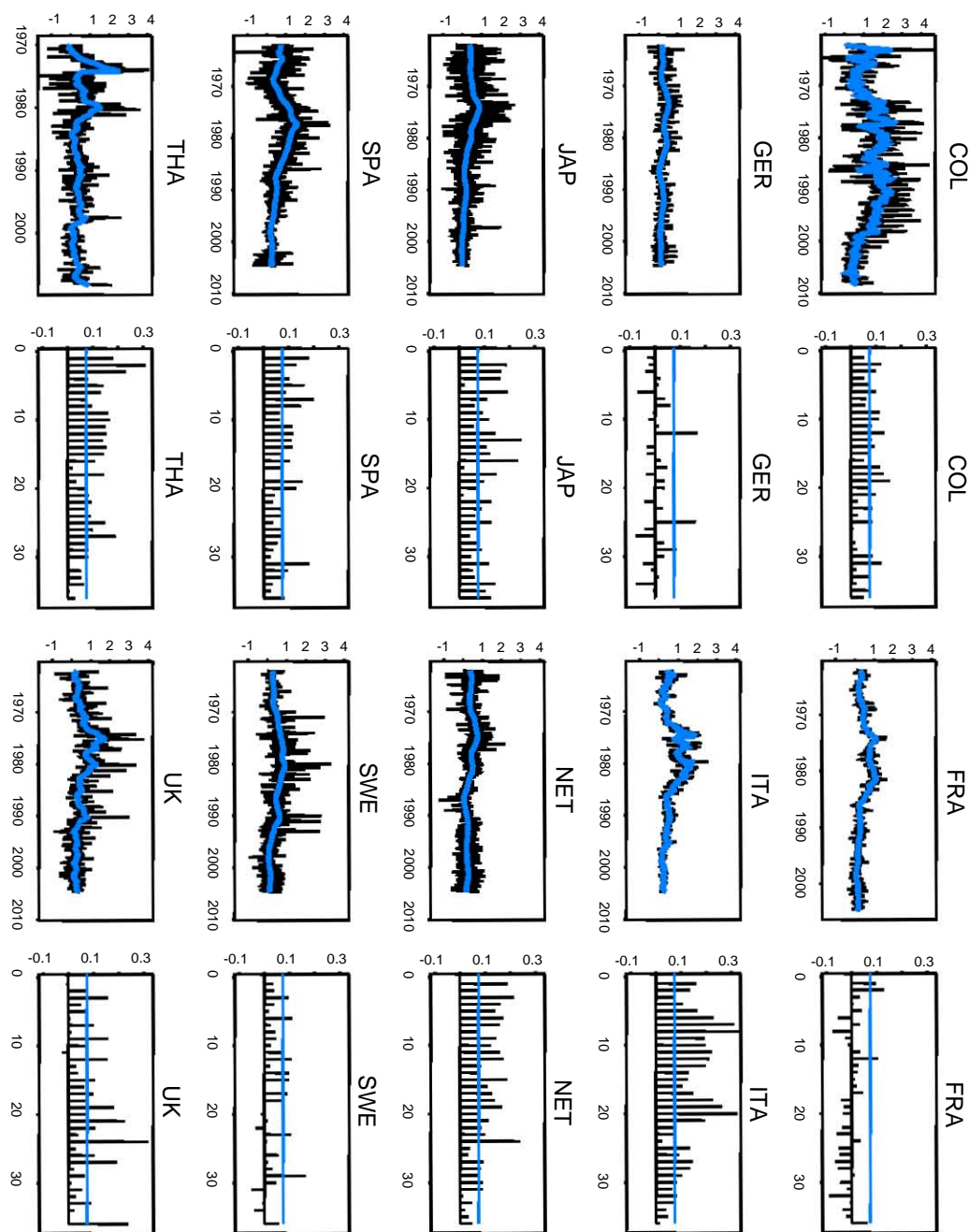


Figure 4: Inflation series for ten countries and estimated trend, together with the differences between the autocorrelations of $(\Delta_{12}y_t)^2$ and the squared autocorrelations of $\Delta_{12}y_t$.

	COL	FRA	GER	ITA	JAP	NET	SPA	SWE	THA	UK
$\hat{\sigma}_\varepsilon^2$	0.2211	0.0226	0.0403	0.0338	0.1855	0.0408	0.2069	0.1198	0.2523	0.0645
$\hat{\sigma}_\eta^2$	0.0593	0.0021	0.0006	0.0068	0.0010	0.0012	0.0029	0.0008	0.0129	0.0082
$\hat{\sigma}_w^2$	0.0001	0.0006	0.0013	0.0005	0.0021	0.0136	0.0077	0.0035	0.0001	0.0032
Mean ($\hat{\nu}_t$)	-0.003	-0.010	-0.001	0.006	-0.029	-0.024	-0.020	-0.006	0.054	0.006
SK($\hat{\nu}_t$)	-0.054	0.036	0.337*	0.506*	0.392*	0.195	0.421*	0.156	0.124	0.339*
$\kappa(\hat{\nu}_t)$	5.631*	4.198*	3.926*	4.533*	4.274*	4.615*	4.649*	5.161*	4.260*	4.035*
$r_2^\nu(1) - [r^\nu(1)]^2$	0.0837*	0.1376*	0.0822*	0.1387*	0.126*	0.0547	0.2053*	0.0794*	0.1161*	0.0147
$r_2^\varepsilon(1) - [r^\varepsilon(1)]^2$	0.1629*	0.0960*	0.1018*	0.1671*	0.0687	0.0769*	0.1427*	0.0661	0.0400	-0.0260
$r_2^\eta(1) - [r^\eta(1)]^2$	0.1205*	0.1007*	-0.0318	0.1214*	0.0493	-0.0158	0.0207	-0.0440	0.0711	0.0260
$BP_\nu(24)$	67.0274*	40.9241*	34.9657	183.3658*	176.4537*	159.3409*	174.7435*	41.4041*	122.1610*	48.5088*
$BP_\varepsilon(24)$	71.9313*	46.7868*	41.2224*	261.6245*	157.6137*	156.9164*	163.3103*	32.9293	121.5432*	39.9723*
$BP_\eta(24)$	43.8793*	262.4986*	95.7117*	193.4089*	65.4236*	104.5669*	376.8511*	77.7295*	187.0932*	147.0624*
LM	10.2306*	19.3536*	15.7767*	9.2716*	7.9971*	29.171*	3.8801	0.7012	5.0621	24.0882*

* Significant at 5%; SK: Skewness; κ : Kurtosis; $r(h)$: Correlation of order h .

Table 3: Estimates of the parameters of a random walk plus noise model with stochastic seasonality and summary statistics of the corresponding innovations $\hat{\nu}_t$ for inflation series.

3 also reports several sample moments of the estimated innovations. We can observe that they still have leptokurtic distributions although the kurtosis coefficients are smaller than in the original data. Furthermore, Table 3 reports the differences of order one between the autocorrelations of $\hat{\nu}_t^2$ and the squared autocorrelations of $\hat{\nu}_t$ as well as for the auxiliary residuals. Taking into account that under conditional homoscedasticity the distribution of these differences can be approximated by a $N(0, 1/T)$, we have marked the differences which are significantly larger than zero. All countries except United Kingdom show symptoms of heteroscedasticity. It is interesting to know that even in United Kingdom, the differences between autocorrelations corresponding to seasonal orders are significantly larger than zero.

To identify which component could be causing the conditional heteroscedasticity, Figure 6 represents the differences between the autocorrelations of the squared auxiliary residuals and the corresponding squares of the autocorrelations for $\hat{\varepsilon}_t$ and $\hat{\eta}_t$, respectively. When looking at these differences, we observe that in Colombia, France, Thailand and United Kingdom, they are larger in the long-run residuals. For all the other countries, the dynamics of the short-run residuals are stronger.

Consequently, the Q-STARCH model is fitted to each of the series of inflation. In all series, the seasonal component is homoscedastic. The long-run noise is also homoscedastic in all series but Colombia, France, Thailand and United Kingdom. Finally, in these four series the transitory component is homoscedastic while for all the others, it is heteroscedastic. Table 4 reports the estimated parameters. As expected given our previous results on the tests based on the differences of auto-

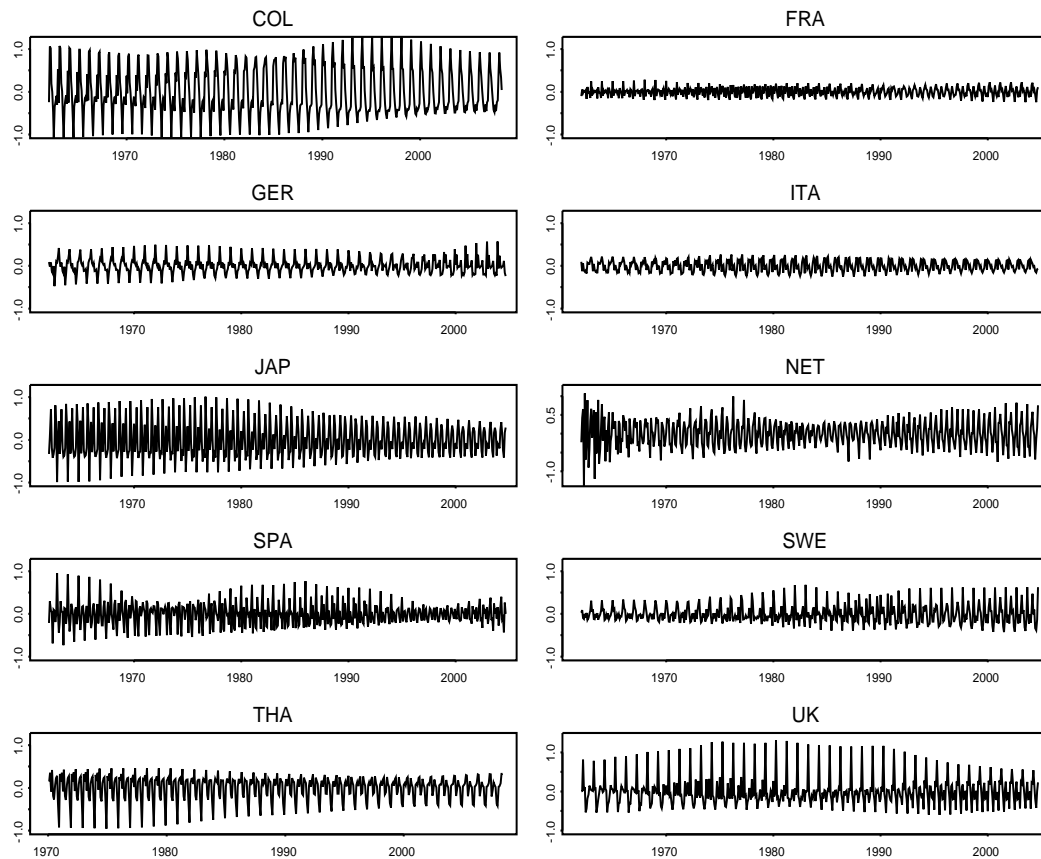


Figure 5: Seasonal component (stochastic seasonality) for the inflation series of ten countries.

correlations, the ARCH coefficients are significant for all countries. Note that, as it is usual in financial time series, the persistence estimated for the GARCH models is very close to unity running from 0.72 in Japan to 0.99 in Colombia, the Netherlands and Thailand. Finally, with respect to the estimated asymmetry parameters, we can observe that they are positive and significant in Colombia, France, Germany, Italy, Sweden, Thailand and United Kingdom while they are negative and not significant in Japan, the Netherlands and Spain. Therefore, our results support the Friedman hypothesis of larger inflation increasing future uncertainty in the former set of countries while the uncertainty of inflation in Japan, the Netherlands and Spain is time-varying although it does not depend on past levels of inflation.

Finally, Table 4 also represents the summary statistics of the standardized in-

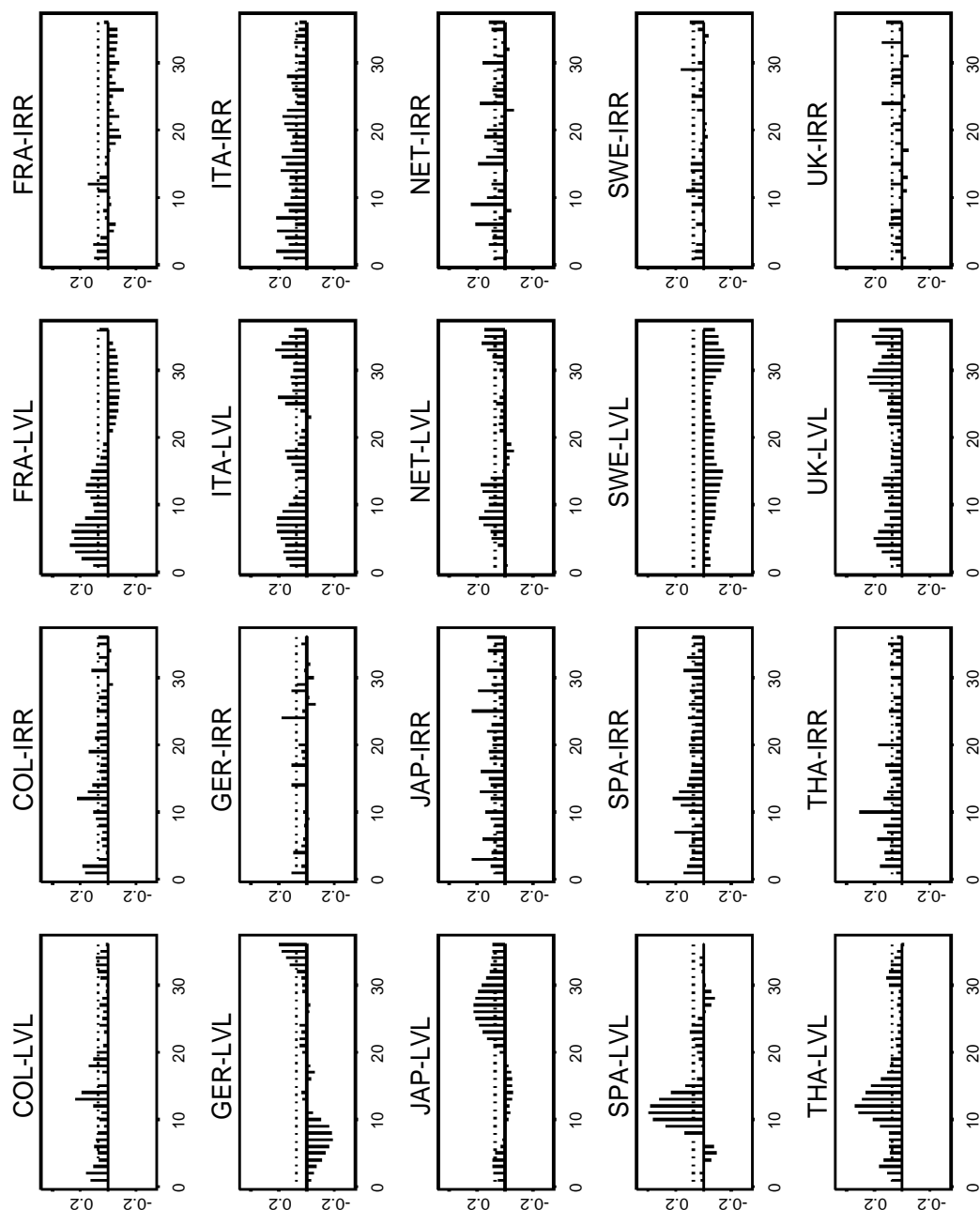


Figure 6: Differences between the autocorrelations of squares and squares of the autocorrelations of the auxiliary residuals for the inflation series of ten countries.

	COL	FRA	GER	ITA	JAP	NET	SPA	SWE	THA	UK
α_0	0.0308 (2.0229)	0.0347 (10.0281)	0.0041 (1.2628)	0.0003 (0.9606)	0.0014 (1.0558)	0.00001 (0.9257)	0.0040 (1.4281)	0.0056 (1.3095)	0.00001 (0.4390)	0.00001 (1.1150)
α_1			0.1031 (1.9832)	0.0556 (2.2321)	0.1196 (2.0663)	0.1567 (1.2190)	0.1398 (2.6973)	0.0463 (1.6209)		
α_2			0.8770 (13.7676)	0.9418 (36.0084)	0.8778 (14.8065)	0.8431 (6.5480)	0.8280 (13.6756)	0.9014 (14.5196)		
α_3			0.0412 (2.0041)	0.0082 (1.0522)	0.0259 (1.1344)	0.0095 (0.6260)	0.0061 (0.2520)	0.0323 (2.0718)		
γ_0	0.0002 (1.7072)	0.0001 (0.9450)	0.0005 (4.0950)	0.0030 (14.7920)	0.0006 (5.7859)	0.0008 (6.3349)	0.0011 (8.7665)	0.0006 (8.0000)	0.0032 (4.0507)	0.0001 (1.4780)
γ_1	0.2504 (17.7511)	0.4413 (8.1825)							0.3104 (10.2646)	0.2990 (8.9308)
γ_2	0.7491 (53.0036)	0.5067 (9.5535)							0.6842 (22.8943)	0.7001 (20.8411)
γ_3	0.0147 (6.4695)	0.0122 (1.9572)							0.0633 (22.5000)	0.0131 (11.8506)
σ_ω^2	0.0098 (12.5366)	0.0008 (7.1271)	0.0037 (8.3849)	0.0005 (6.1968)	0.0069 (15.5946)	0.0095 (14.8151)	0.0047 (8.3973)	0.0038 (7.8182)	0.0294 (9.0676)	0.0193 (14.5179)
$LogL$	34.2307	487.6183	279.9152	425.8460	142.6143	286.2358	98.9262	226.8176	50.0043	222.9343
Mean ($\hat{\nu}_t$)	0.0423	-0.089	-0.053	-0.082	-0.104	-0.137	-0.090	-0.033	0.0751	0.043
SK ($\hat{\nu}_t$)	0.2773	-0.490*	-0.840*	-0.454*	0.875*	-0.080*	-0.055*	0.148*	0.4988*	0.242*
κ ($\hat{\nu}_t$)	4.5010*	5.780*	6.227*	6.238*	10.709*	5.888*	3.904*	5.068*	4.4337*	3.912*
$r_2^\nu(1) - [r^\nu(1)]^2$	0.1602*	0.0638	0.0033	-0.0015	0.0127	0.0290	-0.0180	0.0585	0.0316	0.0151
$BP_\nu(24)$	40.7960*	15.5213	15.3395	34.5872	37.8644*	34.6800	54.8816*	33.0441	26.5590	32.0397
LM	16.1498*	11.0866*	1.3274	2.9751	0.4480	1.8018	0.8113	6.5046*	6.5490*	6.6906*

* Significant at 5%; SK: Skewness; κ : Kurtosis; $r(h)$: Correlation of order h .

Table 4: Estimates of the Q-STARCH model with stochastic seasonality for inflation series and summary statistics of the innovations $\hat{\nu}_t$ for inflation series.

novations ν_t of the eight series of inflation. We can observe that the differences between the autocorrelations of squares and the squared autocorrelations are no longer significant except for Colombia, Japan and Spain. In the latter series, the seasonal correlation is significant. It is possible that the seasonal component of the Spanish inflation series may have some kind of heteroscedastic behavior. The extension of the model to incorporate a conditional heteroscedastic seasonal component is left for further research. Finally, note that using the LM statistic, the inflations of Colombia, France, Sweden, Thailand and United Kingdom are still heteroscedastic. However, as we have seen in previous sections, the behaviour of the LM test is worse than for the $BP_\nu(24)$ test. Consequently, our final conclusions are based on the latter test.

5 Conclusions

In this article, we fit a seasonal unobserved components model to monthly series of inflation. The model allows the transitory and long run components to be conditionally heteroscedastic. In particular, the variances of the unobserved noises are modelled as QGARCH processes. We first show how to use the auxiliary residuals to identify which components are heteroscedastic. We carry out Monte Carlo experiments to show that, if a component is homoscedastic, the finite sample distribution of the differences between the autocorrelations of the corresponding squared residuals and the squared autocorrelations of the residuals can be adequately approximated by a Normal distribution with zero mean and variance $1/T$. However, when at least one of the components is heteroscedastic, these differences have means different from zero and, consequently, the heteroscedasticity can be detected by looking at them. We propose to use these differences not only with estimated innovations but also with the auxiliary residuals. Our results also show that using auxiliary residuals to detect conditional heteroscedasticity increases the power with respect to detecting the heteroscedasticity using the estimated innovations. However, the transmission of heteroscedasticity between components may distort the correct identification of the heteroscedastic component. Further research of measuring this transmission is worthwhile.

Finally, the model is fitted to analyze the dynamic behaviour of inflation in ten developed and emerging countries. The auxiliary residuals show that, in some of the countries, when there is heteroscedasticity, it affects the transitory component, while the uncertainty of the long-run component is constant. In Colombia, France, Thailand and United Kingdom, the long-run component is heteroscedastic. In all cases, the variance of the heteroscedastic component of inflation can be represented by a QGARCH model. With the exception of Japan, the Netherlands and Spain, all

the countries with time-varying uncertainty show a positive relationship between the uncertainty and past levels of inflation, supporting the Friedman hypothesis of uncertainty of inflation increasing with its level.

References

- [1] Baillie, R.T., C. Chung and M. Tieslau (1996): "Analyzing inflation by the fractionally integrated ARFIMA-GARCH model," *Journal of Applied Econometrics*, 11, 23-40.
- [2] Ball, L. (1992): "Why does high inflation rise inflation-uncertainty?," *Journal of Monetary Economics*, 29, 371-388.
- [3] Ball, L. and S.G. Cecchetti (1990): "Inflation and Uncertainty at Short and Long Horizons," *Brooking Papers on Economic Activity*, 215-254.
- [4] Bhar, R. and S. Hamori (2004): "Empirical characteristics of the permanent and transitory components of stock returns: analysis in a Markov switching heteroscedastic framework," *Economics Letters*, 82, 157-165.
- [5] Bollerslev, T. (1986): "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307-327.
- [6] Bos, C. and N. Shephard (2006): "Inference for Adaptive Time Series Models. Stochastic Volatility and Conditionally Gaussian State Space Form," *Econometric Reviews*, 25, 219-244.
- [7] Brandt, M.W. and Q. Kang (2004): "On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach," *Journal of Financial Economics*, 72, 217-257.
- [8] Breidt, F.J. and R.A. Davis (1992): "Time-Reversibility, Identifiability and Independence of Innovations for Stationary Time Series," *Journal of Time Series Analysis*, 13, 377-390.
- [9] Brunner, A.D. and G.D. Hess (1993): "Are Higher Levels of Inflation Less Predictable? A State Dependent Conditional Heteroskedasticity Approach," *Journal of Business & Economic Statistics*, vol. 11, 2, 187-197.
- [10] Broto, C. and E. Ruiz (2006): "Unobserved Component models with asymmetric conditional variances," *Computational Statistics and Data Analysis*, 50, 2146-2166.

- [11] Caporale, T. and B. McKierman (1997): "High and variable inflation: further evidence on the Friedman hypothesis," *Economics Letters*, 54, 65-68.
- [12] Conrad, C. and M. Karanasos (2005): "On the inflation-uncertainty hypothesis in the USA, Japan and the UK: a dual long memory approach," *Japan and the World Economy*, 17, 327-343.
- [13] Cosimano, T. and D. Jansen (1988): "Estimates of the variance of US inflation based upon the ARCH model," *Journal of Money, Credit and Banking*, 20, 409-421.
- [14] Cukierman, A. (1992): *Central Bank Strategy, Credibility, and Independence*. MIT Press, Cambridge.
- [15] Cukierman, A. and A. Meltzer (1986): "A theory of ambiguity, credibility, and inflation under discretion and asymmetric information," *Econometrica*, 54, 1099-1128.
- [16] Cukierman A. and P. Wachtel (1979): "Differential Inflationary Expectations and the Variability of the Rate of Inflation: Theory and Evidence," *American Economic Review*, 69, 3, 444-447.
- [17] Daal, E., N. Naka and B. Sánchez (2005): "Re-examining inflation and inflation uncertainty in developed and emerging countries," *Economics Letters*, 89, 180-186.
- [18] Durbin, J. and S.J. Koopman (2001): *Time Series Analysis by State Space Methods*. Oxford University Press.
- [19] Engle, R.F. (1982): "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," *Econometrica*, 50, 987-1008.
- [20] Engle, R.F. (1983): "Estimates of the variance of the U.S. Inflation Based Upon the ARCH Model," *Journal of Money, Credit and Banking*, 15, 286-301.
- [21] Evans, M. (1991): "Discovering the Link between Inflation Rates and Inflation Uncertainty," *Journal of Money, Credit and Banking*, 23, 2, 169-184.
- [22] Evans, M. and P. Wachtel (1993): "Inflation Regimes and the Sources of Inflation Uncertainty," *Journal of Money, Credit and Banking*, 25, 2nd part, 475-511.

- [23] Foster, E. (1978): "The Variability of Inflation," *The Review of Economics and Statistics*, 60, 3, 346-350.
- [24] Fountas, S., M. Karanasos and M. Karanassou (2000): "A GARCH model of inflation and inflation uncertainty with simultaneous feedback," *Working Papers 414, Queen Mary, University of London, Department of Economics*.
- [25] Friedman, M. (1977): "Nobel Lecture: Inflation and Unemployment," *Journal of Political Economy*, 85, 451-472.
- [26] Garcia, R. and P. Perron (1996): "An analysis of the real interest rate under regime shifts," *Review of Economics and Statistics*, 78, 111-125.
- [27] Grier, K. and M. Perry (1998): "On inflation and inflation uncertainty in the G7 countries," *Journal of International Money and Finance*, 17, 671-689.
- [28] Grier, K. O.T. Henry, N. Olekalns and K. Shields (2004): "The asymmetric effects of uncertainty on inflation and output growth," *Journal of Applied Econometrics*, 19, 551-565.
- [29] Harvey, A.C. (1989): *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press.
- [30] Harvey, A.C. and S. Koopman (1992): "Diagnostic Checking of Unobserved-Components Time Series Models," *Journal of Business & Economic Statistics*, 10, 377-389.
- [31] Harvey, A.C., E. Ruiz and E. Sentana (1992): "Unobserved Component Time Series Models with ARCH Disturbances," *Journal of Econometrics*, 52, 129-157.
- [32] Hwang, Y. (2001): "Relationship between inflation rate and inflation-uncertainty," *Economics Letters*, 73, 179-186.
- [33] Joyce, M. (1995): "Modelling UK Inflation Uncertainty: the Impact of News and the Relationship with Inflation," *Bank of England Working Paper 30*.
- [34] Kim, C.J. (1993): "Unobserved-Component Time Series Models with Markov-Switching Heteroskedasticity: Changes in Regime and the Link between Inflation Rates and Inflation Uncertainty," *Journal of Business and Economic Statistics*, 11, 341-349.
- [35] Kim, C.-J. and C.R. Nelson (1999): *State-Space Models with Regime Switching*. The MIT Press, Cambridge.

- [36] Kontonikas, A. (2004): "Inflation and inflation uncertainty in the United Kingdom, evidence from GARCH modelling," *Economic Modelling*, 21, 525-543.
- [37] Koopman, S.J. and C.S. Bos (2004): "State Space models with a common stochastic variance," *Journal of Business & Economic Statistics*, 22, 346-357.
- [38] Koopman S.J., Harvey, A.C., Doornik, J.A. and Shephard, N. (2000): *STAMP: Structural Time Series Analyzer, Modeller and Predictor*. London: Timberlake Consultants Press.
- [39] Maravall, A. (1987): "Minimum Mean Squared Error Estimation of the Noise in Unobserved Component Models," *Journal of Business & Economic Statistics*, 5, 115-120.
- [40] Okun, A. (1971): "The Mirage of Steady State Inflation," *Brookings Papers on Economic Activity*, 1971-2, 485-98.
- [41] Ord, J.K., A.B. Koehler and R.D. Snyder (1997): "Estimation and Prediction for a Class of Dynamic Nonlinear Statistical Models," *Journal of the American Statistical Association*, 92, No. 440, 1621-1629.
- [42] Palma, W. and M. Zavallos (2004): "Analysis of the correlation structure of square time series," *Journal of Time Series Analysis*, 25, 529-550.
- [43] Sentana, E. (1995): "Quadratic ARCH models," *Review of Economic Studies*, 62, 639-61.
- [44] Stock, J.H. and W. Watson (2007): "Why has US inflation become harder to forecast?," *Journal of Money, Credit and Banking*, 39, 3-33.
- [45] Wells (1996): *The Kalman filter in Finance*. Kluwer Academic Publishing, Amsterdam.